

Enhanced Signatures for Disoriented Chiral Condensates

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The probability distribution in R , the proportion of neutral pions to all pions emitted in heavy ion collisions in certain kinematic regions, has been suggested as a signal of a disoriented chiral condensate (D χ C). Here we note that the signature can be greatly enhanced by making suitable cuts in the data. In particular, we consider reducing the data set such that the k pions with lowest p_T are all neutral. We find that, given such cuts, $\langle R \rangle$ can be substantially different from $1/3$. For example, for a single D χ C domain without contamination due to incoherently emitted pions, $\langle R \rangle$ is $3/5$ given the pion with lowest p_T is neutral, and $5/7$ given the two pions with lowest p_T are both neutral, *etc.*. The effects of multi-domain D χ C formation and noise due to incoherent pion emission can be systematically incorporated. Potential applications to experiments and their limitations are briefly discussed.

I. INTRODUCTION

Recently, there has been growing interest about the possibility of the formation of a disoriented chiral condensate (D χ C) in heavy ion collisions [1–21]. In an ultrarelativistic heavy ion collision, some region may thermalize at a temperature high enough so that chiral symmetry is restored in the region. As the system cools sufficiently rapidly back through the transition temperature, the chiral restored state is unstable as small fluctuations in any chiral direction $(\sigma, \vec{\pi})$ will grow exponentially. This can create regions where the pion field has a macroscopic occupation number. It should be stressed that this scenario is not derivable directly from the underlying theory of QCD and contains a number of untested dynamical assumptions, principally that the cooling is rapid. While the failure of the system to form a D χ C cannot be used to rule out that the system has reached the chiral restoration temperature (as the scenario described above is not derivable directly from QCD), observation of D χ C formation would be a clear signal for chiral restoration at high temperature.

There are clear signatures of D χ C formation provided a single large domain is formed, containing a large number of pions. For example, one expects an excess in low p_T pion production as the characteristic momentum of a pion from a large region is small. Such a signal works even if multiple regions of D χ C are formed provided each region is large, but it is not decisive since one could imagine some other collective low energy effects which produce low p_T pions. On the other hand, since the pions formed in a D χ C, being essentially classical, form a coherent state, this coherent state has some orientation in isospace, and all of the pions in the domain are essentially maximally aligned (given the constraints of quantum mechanics) and point in the same isospin direction. If there are a large number of pions in the domain, this implies a distinctive distribution of R , the ratio of neutral to total pions in the domain [1–4,6–8,18]:

$$f_0(R) = \frac{1}{2\sqrt{R}}. \quad (1)$$

In contrast, the distribution from uncorrelated emissions is narrowly peaked about $1/3$ with a variance, $\langle R^2 \rangle - \langle R \rangle^2 = \frac{2}{9N}$ where N is the total number of pions, and approaches a delta function at $R = 1/3$ when $N \rightarrow \infty$. Since these two distributions are dramatically different, this provides a clear signature for single domain D χ C formation, provided one can kinematically separate the pions from the D χ C from other pions in the system.

Unfortunately, this signature depends critically on the assumption that a single large domain of D χ C is formed, which is *a priori* rather unlikely. One expects the domains are of characteristic size τ , the exponential growth time of the pion fluctuation in the unstable chiral restored state [13,14]. Since the size of the fireball is much larger than typical QCD length scales, it seems unlikely that τ would be of the size of the fireball, and thus formation of a single D χ C domain is improbable. Formation of multiple domains of D χ C, however, tends to wash out the R distribution described by Eq. (1). As the pions emerging from different domains cannot be distinguished kinematically, by the central limit theorem the R distribution will approach a normal distribution peaked at $R = 1/3$. This normal distribution may be distinguished from the normal distribution arising from uncorrelated emission; the case of multiple domains of D χ C will have a substantially larger variance.

Unfortunately, there is an important practical limitation which makes it difficult to exploit the R distribution as a signature. Even under the most optimistic of scenarios, the total number of pions coming from $D\chi C$'s will be a small fraction of the total number of pions. If one includes all pions produced in the reaction, the signal due to the pions from the $D\chi C$ will presumably be overwhelmed. One can apply a low p_T cut to suppress the noise due to incoherently emitted pions. However, even with the low p_T cut, the noise may still be severe, as both the signal and the noise peak at $R = 1/3$. In this paper, we suggest cuts which may dramatically enhance the signal-to-noise ratio. We study the conditional probability distribution of R given only for the events in which the k pions with the lowest p_T are all neutral, and we will show that the expectation value of R is shifted away from $1/3$. Since incoherent emission will result in a very narrow peak around $R = 1/3$, any such shifts should be easily observable. Moreover, one can make successive cuts by increasing the value of k , and enhance the signal in each successive step.

This paper is organized as follows. We will start with the simple scenario and study single domain $D\chi C$ formation in Sec. II. In Sec. III the effects of multi-domain formation and the noises due to incoherently emitted pions will be studied, while potential experimental application and limitations will be discussed in Sec. IV.

II. DOMAIN WITH NON-ZERO ISOSPIN

We are going to start with an unrealistic simple scenario and make it more realistic later on in our discussion. We will consider a single domain which is described by an isosinglet density matrix. (In general, a $D\chi C$ is not a pure state; the wavefunction of the pion coherent state at the core of the “fireball” of the heavy ion collision is entangled with the energetic emission at the edge of the “fireball”.) Such a state can be written as

$$\rho = \sum_{n,I,I_z} c_{nI} \frac{(-1)^{I_z}}{(2I+1)} |n, I, I_z\rangle \langle n, I, I_z|, \quad (2)$$

with $\sum_{n,I} c_{nI} = 1$. Note that the coefficients c_{nI} are real, positive and do not depend on I_z (as the full state is assumed to be isoscalar). The probability distribution of such a mixed state in the isospace is ¹

$$d^2P(\theta, \phi) = \sum_{n,I} c_{nI}/(2I+1) \sum_{I_z} |Y_{II_z}(\theta, \phi)|^2 \sin\theta d\theta d\phi = \sin\theta d\theta d\phi/4\pi, \quad dP(\theta) = \sin\theta d\theta/2, \quad (3)$$

where the angles (θ, ϕ) are defined such that a unit vector in isospace is $(r_x, r_y, r_0) = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$. The probability distribution is uniform, as the condensate is equally likely to point at any direction on the two-dimensional sphere, as demanded by isospin symmetry.

The number operator of neutral pions in the condensate is given by

$$n_0 = a_0^\dagger a_0 = \vec{a}^\dagger \cdot \vec{a} \cos^2\theta = \langle n \rangle \cos^2\theta, \quad (4)$$

with $\vec{a} = (a_x, a_y, a_0)$ is a vector of hermitian annihilation operators which annihilate $\pi_x = (\pi_+ + \pi_-)/\sqrt{2}$, $\pi_y = (\pi_+ - \pi_-)/\sqrt{2}i$, and π_0 , respectively, and $\langle n \rangle = \sum_{n,I} c_{nI}n$ is the expected number of pions. Since $R = n_0/\langle n \rangle$ ², one can easily calculate the probability distribution of R .

$$dP = \frac{1}{2} \sin\theta d\theta = \frac{1}{2} d\cos\theta = d\sqrt{R} = \frac{1}{2} R^{-1/2} dR. \quad (5)$$

By defining $dP \equiv f_0(R) dR$ (the subscript “0” stands for $I = 0$), one has, in the limit that $\langle n \rangle$ is large,

$$f_0(R) = \frac{1}{2} R^{-1/2}, \quad (6)$$

recovering Eq. (1). The distribution is plotted in Fig. 1a. It is obvious that the shape is qualitatively different from the Poisson–Gaussian distribution due to incoherent emissions. The expectation value of R is $1/3$,

$$\langle R \rangle_0 \equiv \int_0^1 R f_0(R) dR = 1/3, \quad (7)$$

¹Here we are assuming that the typical I is much less than $\langle n \rangle$.

² We are assuming that $\langle n \rangle \gg 1$.

which has the simple interpretation that it is equally likely for the pion to be a π_0 , π_+ or π_- , and hence on average a third of the pions are neutral.

This distribution is a consequence of the fact that we have assumed the $D\chi C$ to be an isosinglet, a reasonable assumption on physical grounds. However, let's consider the distribution of R after the $D\chi C$ emits a single π_0 . The density matrix after the emission, which can be written as $\lambda a_0 \rho a_0^\dagger$, where λ is a normalization constant, ρ is the density matrix defined in Eq. (2) and a_0 annihilates a neutral pion. This new density matrix is not an isosinglet. It is straightforward to show that the probability distribution for this state is

$$dP = \frac{\frac{1}{2} \sin \theta \cos^2 \theta d\theta}{\int \frac{1}{2} \sin \theta \cos^2 \theta d\theta} = \frac{\frac{1}{2} R \cdot R^{-1/2} dR}{\int_0^1 \frac{1}{2} R \cdot R^{-1/2} dR} \quad (8)$$

or equivalently,

$$f_1(R) \equiv f(R | \text{1st pion is neutral}) = R f_0(R) \Big/ \int_0^1 R f_0(R) dR = \frac{3}{2} R^{1/2}. \quad (9)$$

The distribution $f_1(R)$ is plotted in Fig. 1b, which is drastically different from $f_0(R)$. The distribution is skewed towards the high end, while $f_0(R)$ is skewed towards the low end. Moreover, the expectation value of R is clearly pushed up:

$$\langle R \rangle_1 \equiv \int_0^1 R f_1(R) dR = 3/5. \quad (10)$$

So we have arrived at the intriguing conclusion that, if the “first pion” emitted from a isosinglet $D\chi C$ is neutral, 60% of the pions subsequently emitted from the $D\chi C$ are neutral, a huge enhancement from the original expectation of 33%.

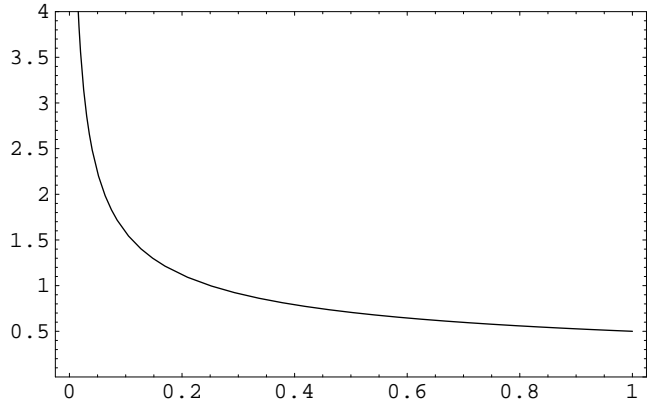


Fig. 1a

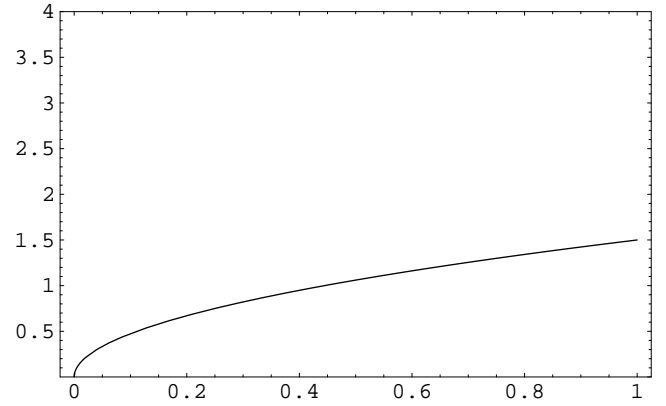


Fig. 1b

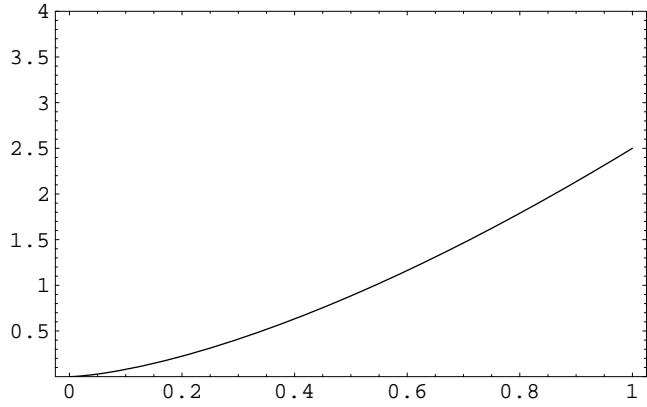


Fig. 1c

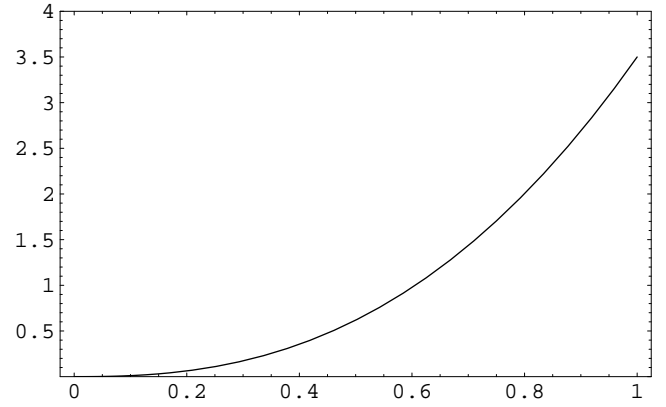


Fig. 1d

FIG. 1. The probability distribution functions $f_k(R)$ for different values of k . Fig. 1a, b, c and d are $f_0(R)$, $f_1(R)$, $f_2(R)$ and $f_3(R)$, respectively.

This extraordinary statement certainly deserves more discussion. First, what is our criterion to decide which is the “first pion”? The answer is simple: it can be any criterion. It does not matter as long as it is *a priori* equally likely to be a π_0 , a π_+ or a π_- . The derivation just depends on our removing a neutral pion from the isosinglet $D\chi C$. It can be the first pion emitted in time, or the last one emitted in time, or even the 17th emitted in time. The criterion can also be unrelated to the order of emission. For example, we can choose the “first pion” to be the one with the smallest polar angle. One can use any of these criteria to identify the “first pion”, and if it turns out to be neutral, then the R distribution of subsequent emissions is always given by $f_1(R)$, provided we are in the large number limit. However, this is only true in this idealized scenario, when all the pions are coming from a simple $D\chi C$ domain. In reality, some of the pions come from incoherent emission, and if the “first pion” turns out to be incoherently emitted, the expectation value of R of the remaining pions is still going to be $1/3$, not $3/5$. As a result, we want to choose our criterion in such a way that the “first pion” is likely to originate from the $D\chi C$ and not from incoherent emissions. Since $D\chi C$ pions by hypothesis have low $p_T \sim 1/L$, where L is the size of the domain, a natural choice is to use the pion with the lowest p_T as our “first pion”.

After clarifying the meaning of the term “first pion”, we move on to discuss the physical origin of the modification of the probability distribution of R . In a nutshell, we are seeing the physics of (iso)spin alignment due to Bose condensation. To illustrate the point, let us first consider the following apparently unrelated Stern–Gerlach experiment. Consider a large number of massive spin-1 particles, which for concreteness will be called deuterons. Initially they are all polarized along a randomly chosen direction \vec{n} , which is *a priori* equally likely to be any direction in three dimensional space. In other words, $\vec{S} \cdot \vec{n} = 0$ for all the deuterons. Now let us pick one of these deuterons and pass it through a Stern–Gerlach spectrometer which measures S_z , the spin along the z -axis. What is the probability that the measurement gives $S_z = 0$? The answer is clearly $1/3$, as the cases for $S_z = +1, 0$ and -1 are equally likely. On the other hand, if the measurement on the first deuteron gives $S_z = 0$, what is the conditional probability for the next deuteron to pass through the Stern–Gerlach spectrometer also to be measured to have $S_z = 0$? The answer this time is no longer $1/3$. The spins of all the deuterons are aligned along the same direction \vec{n} , and that the first deuteron is measured to have $S_z = 0$ suggests \vec{n} is more probable to be more or less aligned along \vec{z} than otherwise. As a result, the conditional probability is no longer $1/3$, but can be easily shown to be $3/5$, which is exactly the predicted value for $\langle R \rangle_1$ in Eq. (10). The situation for a single domain of $D\chi C$ is analogous, with isospin aligned pions instead of spin aligned deuterons. By construction, the pions in a $D\chi C$ domain are isospin aligned, and by the same analysis, we have shown that the knowledge of the “first pion” being neutral can dramatically modify the conditional probability distribution of R .

One can also consider the conditional probability distribution of R in the case that the “first pion” is charged. Note that

$$f_0(R) = \frac{1}{3}(f(R|\text{1st pion is a } \pi_+) + f(R|\text{1st pion is a } \pi_-) + f(R|\text{1st pion is a } \pi_0)), \quad (11)$$

and hence, since $f_1(R) = f(R|\text{1st pion is a } \pi_0)$,

$$\tilde{f}(R) \equiv f(R|\text{1st pion is charged}) = \frac{3}{2}f_0(R) - \frac{1}{2}f_1(R) = \frac{3}{4}(1 - R)R^{-1/2}. \quad (12)$$

The expectation value of R , given that the “first pion” is charged, can be easily shown to be $1/5$. As a consistency check, one can calculate $\langle R \rangle$, the expectation value of R regardless of the species of the “first pion”. Since the “first pion” is twice as likely to be charged as to be neutral,

$$\langle R \rangle_0 = \frac{1}{3}(\frac{3}{5} + 2 \times \frac{1}{5}) = \frac{1}{3}, \quad (13)$$

agreeing with Eq. (7).

Lastly, we will study the conditional probability distribution of R given that the k pions with the lowest p_T , which will be hereafter referred to as the “first k pions”, are all neutral. It is straightforward to show that in this case

$$dP = \frac{\frac{1}{2} \sin \theta \cos^{2k} \theta d\theta}{\int \frac{1}{2} \sin \theta \cos^{2k} \theta d\theta} = \frac{\frac{1}{2} R^k \cdot R^{-1/2} dR}{\int_0^1 \frac{1}{2} R^k \cdot R^{-1/2} dR}. \quad (14)$$

and

$$f_k(R) \equiv f(R|\text{1st } k \text{ pions are all neutral}) = R^k f_0(R) \Big/ \int_0^1 R^k f_0(R) dR = (k + \frac{1}{2}) R^{k-1/2}. \quad (15)$$

The distributions $f_2(R)$ and $f_3(R)$ are plotted in Fig. 1c and d, respectively. One can see that as k increases, the distribution is more and more skewed towards the high end. As a result, the expectation value of R increases with k .

$$\langle R \rangle_k \equiv \int_0^1 R f_k(R) dR = (2k+1)/(2k+3). \quad (16)$$

It is useful to define Q as the ratio of the number of π_+ to the number of total pions emitted. By symmetry it is also the ratio of the number of π_- to the number of total pions emitted, and since $R + 2Q = 1$,

$$\langle Q \rangle_k = 1/(2k+3). \quad (17)$$

From the above analysis, the prescription to enhance the collective signal is quite clear. One should make successive cuts on the data sample on the condition that the k pions with the lowest p_T are all neutral, and measure $\langle R \rangle_k$ after each cut to see if it increases as predicted in Eq. (16). This result, however, depends on the assumption that we have only a single domain of $D\chi C$ without any contamination due to incoherent pion emissions. Since this assumption is unrealistic for heavy ion collision experiments, the scenario we studied in this section is only an idealized situation. In the next section, we will discuss more realistic scenarios.

III. THE EFFECTS OF MULTI-DOMAIN FORMATION AND INCOHERENT EMISSIONS

The scenario considered in the last section is highly unrealistic in at least two ways. First, as discussed in the introduction, single domain $D\chi C$ formation is highly unlikely. For a realistic treatment one must study $D\chi C$ formation with more than one domain, each pointing in a different direction in the isospace. Moreover, we have neglected the effect of incoherently emitted pions, which have very important effects. If the neutral “first pion” is incoherently emitted, the R distribution of the remaining pions is described by $f_0(R)$, instead of $f_1(R)$ when the “first pion” comes from the $D\chi C$. In this section, we will incorporate these two effects and see how the predictions above are modified.

We will study the expectation value of R , or equivalently the expectation of Q , for a situation described by the following parameters. The coherent fraction χ is the fraction of pions which originate from $D\chi C$ domains, so that when $\chi = 1$, all pions are coherently emitted, and when $\chi = 0$, all pions are incoherently emitted. We will consider the case where there are N domains, all containing an equal number of pions³, which will be assumed to be large. Each domain is described by an isosinglet density matrix, but the isospins of pions in different domains are uncorrelated. Now the question is: if the “first k pions” in this channel are all neutral, what are the expectation values of R and Q among the rest of the pions?

The answer turns out to be the following expression:

$$\langle R \rangle = \frac{1}{3} + 2\Delta, \quad \langle Q \rangle = \frac{1}{3} - \Delta. \quad (18a)$$

The shift Δ is given by

$$\Delta = \chi \sum_{j=0}^k P_j \left(\frac{1}{3} - \frac{1}{2j+3} \right) = \chi \left(\frac{1}{3} - \sum_{j=0}^k P_j \frac{1}{2j+3} \right), \quad (18b)$$

where

$$P_j = \binom{k}{j} p^j (1-p)^{k-j}, \quad p = \chi/N. \quad (18c)$$

Each term in this formula has a simple interpretation:

- The expectation value $\langle R \rangle$ is always $1/3$ for the incoherently emitted pions. Only the pions coming from the domains are affected by isospin alignment; hence the outstanding factor of χ .
- Each coherently emitted pion comes from one of the domains, which will be called domain X. How many of the “first k pions” also come from domain X? The probability for each pion coming from domain X is $p = \chi/N$, and the probability that j of the “first k pions” coming from domain X is $P_j = \binom{k}{j} p^j (1-p)^{k-j}$.
- Given that j of the “first k pions” is coming from domain X, the conditional expectation value of Q decreases from $1/3$ to $1/(2j+3)$, while the conditional expectation value of R increases by twice the above quantity.

³This assumption of all domains having the same number of pions is unrealistic but is made for illustrative purposes. The effects of unequal domain sizes will be briefly discussed below.

In passing, we note that Δ can also be expressed as an integral or the hypergeometric function ${}_2F_1$:

$$\begin{aligned}\Delta &= \chi \left(\frac{1}{3} - \frac{1}{\sqrt{p^3}} \int_0^{\sqrt{p}} dz z^2 (z^2 + 1 - p)^k \right) \\ &= \chi \left(\frac{1}{3} - \frac{\cos^{2k+3} \Theta}{\sin^3 \Theta} \int_0^\Theta d\vartheta \frac{\sin^2 \vartheta}{\cos^{2k+4} \vartheta} \right), \quad \tan^2 \Theta = \frac{\chi/N}{1 - \chi/N} \\ &= \frac{1}{3} \chi \left(1 - (1-p)^k {}_2F_1 \left[\frac{3}{2}, -k, \frac{5}{2}, \frac{-p}{1-p} \right] \right).\end{aligned}\tag{19}$$

In Fig. 2, we have made contour plots of $\Delta = 1/30$ (such that $\langle R \rangle = 0.4$ and $\langle Q \rangle = 0.3$), for $k = 1, \dots, 5$ in the $(\chi, 1/N)$ parameter space. The horizontal axis is the coherent fraction χ while the vertical axis is $1/N$ where N is the number of domains. Both χ and $1/N$ range from 0 to 1. Thus, for example, with 3 domains and $\chi = 0.6$, in order to have $\Delta \geq 1/30$ we must have $k \geq 3$.

Equations (18) illustrate the main results of this paper. One can see that, without any D χ C formation, $\chi = 0$ (corresponding to the left edge of Fig. 2), Δ vanishes, and $\langle R \rangle = \langle Q \rangle = 1/3$ as expected. The bottom edge of the plot corresponds to $N \rightarrow \infty$ and also gives $\Delta = 0$ for any finite value of k . The shift Δ is largest for a single domain of D χ C without any noise due to incoherently emitted pions, *i.e.*, when $\chi = N = 1$ (the top right corner of the contour plots), giving $\Delta = 1/3 - 1/(2k+3)$ and reproducing Eqs. (16) and (17). For fixed values of (χ, N) , Δ increases with k , accounting for the spreading of the parameter space with $\Delta > 0.4$ as k increases from 1 to 5 in Fig. 2.

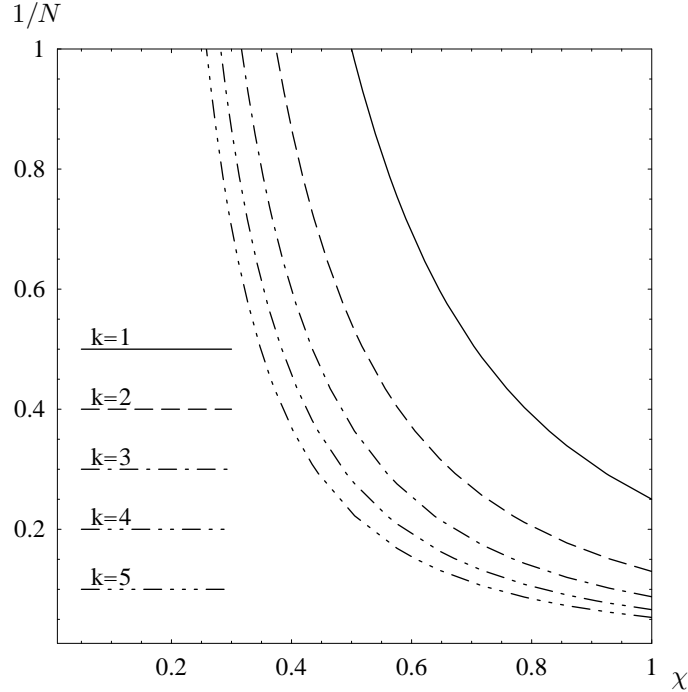


FIG. 2. Contour plots of $\Delta = 1/30$ (such that $\langle R \rangle = 0.4$ and $\langle Q \rangle = 0.3$) for different values of k . The horizontal axis is the coherent fraction χ , while the vertical axis is $1/N$ where N is the number of domains. Both χ and $1/N$ range from 0 to 1. The curves, from top right to bottom left are for $k = 1, 2, 3, 4$ and 5 , respectively. The shift Δ is larger than 0.4 above the curves and smaller than 0.4 below the curves.

One expects that when the number of D χ C domains is large ($N \gg 1$) or when most of the pions are incoherently emitted ($\chi \ll 1$), it will be difficult to observe clear signals of D χ C formation. However, in such situations χ/N is small and Δ is dominated by the $j = 1$ term (the $j = 0$ term always identically vanishes) and

$$\Delta = \frac{2\chi^2 k}{15N} + \mathcal{O}\left(\frac{\chi^3}{N^2}\right).\tag{20}$$

Thus a large k may make up for a small coherent fraction χ , or a large number of domains N , and enhance Δ , which describes the shift of $\langle R \rangle$ and $\langle Q \rangle$ from $1/3$, to an experimentally measurable magnitude. From the form of Eq. (20), one expects this shift to be substantial whenever $k \sim N/\chi^2$. However, even for a value of k as small as $N/4\chi^2$, $\Delta = 1/30 + \mathcal{O}(\chi^3/N^2)$, which translates to $\langle R \rangle = 0.4$ and $\langle Q \rangle = 0.3$ — a substantial deviation from the incoherent case. This suggests one should make successive cuts for events where the k pions with lowest p_T are all neutral, and study $\langle R \rangle$ after each cut. An increase of $\langle R \rangle$ with k would suggest that D χ C domains are formed.

Equation (20) appears to suggest that one can increase Δ to an arbitrarily large magnitude by choosing a sufficiently large value of k . Of course this is not true. Equation (20) is obtained as the leading term in a χ/N expansion, but when $k \rightarrow \infty$, this expansion breaks down as terms of higher order in χ/N are enhanced by factors of $\binom{k}{j}$. We can easily see that

$$\Delta \rightarrow \frac{1}{3}\chi, \quad \langle R \rangle \rightarrow \frac{1}{3} + \frac{2}{3}\chi, \quad \langle Q \rangle \rightarrow \frac{1}{3}(1 - \chi), \quad k \rightarrow \infty \text{ with } \chi \text{ and } N \text{ fixed.} \quad (21)$$

In other words, our signal enhancement scheme is fundamentally limited by the amount of noise due to incoherently emitted pions. When χ is small, most of the pions are incoherently emitted, and for them, $\langle R \rangle$ is always around $1/3$ regardless of what cuts one makes. On the other hand, the large k limit of $\langle R \rangle$ does not depend on N , the number of D χ C domains. Recall that we have several distinctive signatures, like the R distribution in Eq. (1) and the conditional expectation values for R described in the previous section, for a single domain of D χ C, where all the pions in the D χ C are isospin aligned. With multi-domain formation, where the pions in different domains may point to different directions in isospace, the effect of isospin alignment is greatly washed out. However, given that the “first k pions” are all neutral with $k \gg N$, it is probabilistically extremely likely that each of the N domains is the origin of some of these “first k pions”. As a result, each of these N domains are well-aligned along the π_0 direction, and hence also well-aligned with each other. As $k \rightarrow \infty$, the N domains look more and more like a single big domain in the π_0 direction, which is the case where the signature is the most dramatic. In short, with large k , our cuts are picking out the events where the signals are the strongest, and hence resulting in a large signal-to-noise ratio.

In above we have assumed the sizes of all N domains are identical for illustrative purposes. A more realistic treatment would have N domains, each with different sizes p_i , $1 \leq i \leq N$, such that $\sum_i p_i = \chi$. (The size of a domain is defined to be the fraction of pions which originate from this particular domain.) Then Eqs. (18) are generalized to

$$\Delta = \frac{1}{3}\chi - \sum_{i=1}^N p_i \sum_{j=0}^k \binom{k}{j} p_i^j (1-p_i)^{k-j} \frac{1}{2j+3}. \quad (22)$$

In the weak signal limit, *i.e.*, when all the p_i ’s are small, Eq. (20) gets modified to

$$\Delta = \frac{2k}{15} \sum_{i=1}^N p_i^2 + \mathcal{O}(\frac{\chi^3}{N^2}) = \frac{2k}{15} \bar{p} + \mathcal{O}(\frac{\chi^3}{N^2}), \quad (23)$$

where $\bar{p} = \sum_i p_i^2$ has the following nice interpretation: \bar{p} is the average over all pions (both coherently and incoherently emitted) of the sizes of the originating domains, which is p_i for a pion from domain i and zero for an incoherently emitted pion. For N domains of equal sizes, $\bar{p} = \chi^2/N$ and Eq. (20) is recovered. Again we see that Δ grows linearly with k in the weak signal limit. As $k \rightarrow \infty$ the shift Δ is again limited by the bound (21), which applies also for the cases of unequal domain sizes.

IV. SUMMARY

To recapitulate, we suggest the following experimental procedures:

- Count the number of neutral and charged pions *event by event* from heavy ion collision experiments and measure their individual transverse momenta and rapidities.
- Apply a low p_T cut to suppress the noise due to uncorrelated pion emission.
- Bin the events in different rapidity windows.
- In each rapidity window, calculate the expectation value $\langle R \rangle$.
- Make a cut to retain only events where the pion with the lowest p_T is neutral.
- Calculate, in each rapidity window, the expectation value $\langle R \rangle$ for all remaining pions in all events which survive the cut.
- Make another cut on the surviving events to retain only those where the pion with the second lowest p_T is also neutral.

- Again, calculate in each rapidity window the expectation value $\langle R \rangle$ for all remaining pions in all events which survive the cuts.

- Repeat the above prescription of making successive cuts to retain only events in which the pion with the next lowest p_T is also neutral, and calculate $\langle R \rangle$ for each rapidity window after each cut. If we find $\langle R \rangle$ deviates from $1/3$ then we are seeing signatures from $D\chi C$ s.

Note that this prescription requires reconstructions of p_T 's of individual pions, both charged and neutral. We have also presumed that the coherent fraction χ and the number of domains formed N are roughly the same for each event. (More specifically, the probability distributions for χ and N are narrow peaked.)

By applying these successive cuts, we are retaining the events with $D\chi C$ formation *and* most of the pions are well-aligned along the π_0 direction. What is being cut are the events with $D\chi C$ formation but most of the pions are well-aligned along the π_x or π_y directions, and the events where there are incoherent pions with very low p_T , which is the main source of noise to our signal. As a result, these successive cuts are substantially improving the signal-to-noise ratio, making it easier to observe $D\chi C$ formation. On the other hand, just like any other cuts on data to suppress the noises, we are giving up on statistics. Moreover, for large k we are cutting on rare events so the loss in statistics can be severe. For the cases where the signal is weak (small coherent fraction $\chi \ll 1$ or large number of domains $N \gg 1$) on each cut we are losing about two-thirds of the events.

In conclusion, we have devised new cuts to enhance the signal in searches for $D\chi C$. These cuts retain only events where the k pions with lowest p_T are all neutral. We have shown that, after these cuts, the fraction of neutral pions within the remaining sample is substantially larger if $D\chi C$ s are formed in the heavy ion collision.

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